universidad de león

Department of Economics and Statistics

Departamento de Economía y Estadística

Working Papers

Documentos de Trabajo

Inequality decomposition and human development

Ricardo Martínez

WP 2014/01

INEQUALITY DECOMPOSITION AND HUMAN DEVELOPMENT*

Ricardo Martínez†
University of León

July 28, 2014

Abstract

The human development index, which takes into account achievements in health, education, and income, is considered a good measure of the social attainments of a country. The distribution of human development in the world is imbalanced and the degree of cohesion is low. This inequality has varied during recent years. In this paper we present evidence that improvements in the cohesion of human development are mostly attributed to education, whereas health and income have made poor contributions. To do this we exploit the multiplicative structure of the human development index and decomposition of the Theil inequality index.

Keywords: World, human development, inequality, inequality decomposition, Theil index.

JEL Classification: I30, D63

*Financial support from Ministerio de Ciencia under grant ECO2011-29355, Junta de Andalucía under grants SEJ5980 and SEJ4941, and Grupo PAIDI SEJ426 is gratefully acknowledged.

†Departamento de Economía y Estadística, Universidad de León, León 24071, Spain. E-mail: ricardo.martinez@unileon.es. Tel. (+34) 987 29 1911.

1 Introduction

Using Amartya Sen's idea of functioning and capabilities Sen (1985), in 1990 the United Nations (UN) proposed a protocol to measure the overall degree of development of a country, since per capita GDP alone was deemed to be insufficient to illustrate the social achievements of countries. The UN identified health, education, and material wellbeing (per capita GDP) as the keystones of human development, and combined these three indicators into a single measure: the so-called *human development index* (HDI). In 2010, coinciding with the 20th anniversary of its human development reports, the United Nations Development Programme decided to update several aspects of the measurement of human development.[1] The new HDI utilizes life expectancy at birth (LEB) as a proxy for health, the geometric mean between expected years of schooling (EYS) and mean years of schooling (MYS) as a proxy for education, and per capita GNI (in fact the logarithm of per capita GNI) as an indicator of income. The HDI is the geometric mean of these three indicators.[2]

Not all countries exhibit the same level of human development and Pillarisetti (1997) and Martínez (2011) show that the differences among nations with regard to the HDI are very relevant. In particular, Martínez (2011) uses several inequality indices to illustrate the existence of significant disparities among countries and how global cohesion of human development has evolved over the last decades. For example, in 2010 the difference between the most and least developed countries was close to 0.7 (out of a theoretical maximum of 1) and the overall inequality (measured via the Theil index) was 0.05. However, because the HDI comprises three components it is difficult to assess how much of the HDI inequality is explained by inequality for each of these components.

Here we exploit the mathematical properties of the Theil index and the multiplicative structure of the new HDI to provide an answer to this question. More precisely, we decompose the HDI inequality into inequalities for each of its components (health, education, and income). We find that the positive evolution of human development cohesion is mostly due to education, while health has contributed poorly and GNI has been regressive.

The remainder of the paper is organized as follows. In Section 2 we briefly introduce the theoretical tools required for our analysis, including decomposition of the Theil index. In Section 3 we present our main results. Section 4 concludes with some final remarks.

2 Theoretical tools

For each country i, we define its human development index as:

$$d_i = (h_i \cdot e_i \cdot y_i)^{\frac{1}{3}},$$

[1] Despite improvements made in the new version of the HDI, some authors showed that a few drawbacks still persist (see Ravallion (2011); Chakravarty (2011); Klugman et al. (2011); Ravallion (2012); Herrero et al. (2012), for example).

[2] These indicators are normalized to fit between 0 and 1 to make them comparable.

where h_i, e_i, and y_i are the indicators for health, education, and income, respectively. More specifically,

$$h_i = \frac{\text{LEB}_i - 20}{83.6 - 20}, \qquad e_i = \frac{\sqrt{\frac{\text{MYS}_i}{13.3} \cdot \frac{\text{EYS}_i}{20.45}}}{0.952}, \qquad y_i = \frac{\log \text{GNI}_i - \log 100}{\log 59616 - \log 100}.$$

Choices for the maximum and minimum values for these normalizations are in accordance with UNDP (2013). The minimum bounds for LEB, MYS, EYS, and GNI are 20, 0, 0, and 100, respectively. The maximum bounds are the maximum values observed for each component.[3]

Given a vector $x \in \mathbb{R}_{++}$, the Theil inequality index introduced by Theil (1967) is defined as

$$T(x) = \frac{1}{n} \sum_{i=1}^{n} \log\left(\frac{\overline{x}}{x_i}\right),$$

where $\overline{x}$ the mean of x. If we denote by d the HDI vector for all countries ($d = (d_{\text{USA}}, d_{\text{France}}, d_{\text{Japan}}, \ldots)$), we can decompose the inequality $T(d)$ as follows:

$$\begin{aligned}
T(d) &= \frac{1}{n} \sum_{i=1}^{n} \log\left(\frac{\overline{d}}{d_i}\right) \\
&= \frac{1}{n} \sum_{i=1}^{n} \log\left(\frac{\overline{d}}{h_i^{\frac{1}{3}} \cdot e_i^{\frac{1}{3}} \cdot y_i^{\frac{1}{3}}}\right) \\
&= \frac{1}{n} \sum_{i=1}^{n} \log\left(\frac{\overline{h}^{\frac{1}{3}} \cdot \overline{e}^{\frac{1}{3}} \cdot \overline{y}^{\frac{1}{3}}}{h_i^{\frac{1}{3}} \cdot e_i^{\frac{1}{3}} \cdot y_i^{\frac{1}{3}}} \cdot \frac{\overline{d}}{\overline{h}^{\frac{1}{3}} \cdot \overline{e}^{\frac{1}{3}} \cdot \overline{y}^{\frac{1}{3}}}\right) \\
&= \frac{1}{n} \left[\frac{1}{3} \sum_{i=1}^{n} \log\left(\frac{\overline{h}}{h_i}\right) + \frac{1}{3} \sum_{i=1}^{n} \log\left(\frac{\overline{e}}{e_i}\right) + \frac{1}{3} \sum_{i=1}^{n} \log\left(\frac{\overline{y}}{y_i}\right) + \sum_{i=1}^{n} \log\left(\frac{\overline{d}}{\overline{h}^{\frac{1}{3}} \overline{e}^{\frac{1}{3}} \overline{y}^{\frac{1}{3}}}\right)\right] \\
&= \frac{1}{3} T(h) + \frac{1}{3} T(e) + \frac{1}{3} T(y) + \log\left(\frac{\overline{d}}{\overline{h}^{\frac{1}{3}} \overline{e}^{\frac{1}{3}} \overline{y}^{\frac{1}{3}}}\right).
\end{aligned}$$

We can observe that the particular formulation of the Theil index and the multiplicative structure of the HDI allow us to decompose the inequality for human development as a weighted sum of the inequalities for each of the components, plus a residual. This residual can be interpreted as a ratio between the HDI for the mean country and the HDI for a virtual country whose health, education, and income are the corresponding mean for each element. Note that the fact that the HDI is a geometric mean is crucial for this argument. Before 2010 the HDI was an arithmetic mean, so this reasoning would not be possible because the relationship between the inequality of the overall HDI and the inequality for each of its components could not be determined. Selection of the Theil index is also key; other inequality measures are available but they do not provide as clean a decomposition as the Theil index does.

[3] Readers are referred to UNDP (2013) for a more detailed discussion on construction of the HDI.

3 Data and results

All data used in this paper were obtained from the United Nations Development Programme. We analyze data from 1980–2012 and only consider countries with data available for the study period, which represents 89 countries in total.

Figure 1 shows the evolution of the HDI distribution over time. For each year considered we plot the HDI distribution, starting with the country with the highest HDI and ending with the country with the lowest. This plot suggests two ideas. On one hand, global HDI has positively evolved over time. The most developed country in 2010 was better off than the most developed country was in 1980, the second most developed country in 2010 was better off than the second most developed country was in 1980, and so on. These "dominated" improvements have been sustained over time, with very few exceptions (the later the year, the clearer is the improvement). On the other hand, since nations with high and low human development indices may both improve, global HDI cohesion may remain unchanged or even decrease.

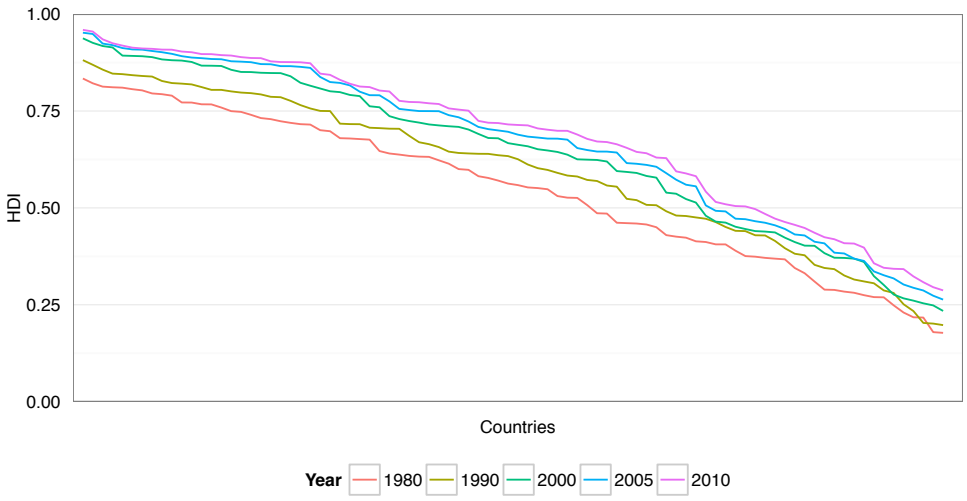

Figure 1: Global HDI distribution during 1980–2010.

Our main findings are summarized in Table 1 and represented in Figure 2. Column 2 in Table 1 and the black line in Figure 2 indicate HDI inequality for the relevant year. Columns 3, 4, and 5, and the green, red, and yellow lines in Figure 2 indicate the Theil inequality indices for health, education, and income, respectively. Finally, the residuals are listed in the last column in Table 1 and denoted by the dashed line in Figure 2.

We can observe from Figure 2 that human development has become more evenly distributed across the world from 1980 to 2012; HDI inequality has been steadily decreasing over these decades, and in 2012 was 37.2% lower than it was in 1980. In absolute terms, the reduction from 1980 to 2012 is 0.0265.

Figure 2 clearly illustrates that not all HDI components contributed equally to this improvement. Apart from slight growth during the 1980s and 1990s, inequality in health has decreased since

Year	$T(d)$	$T(h)$	$T(e)$	$T(y)$	residual
1980	0.0712	0.0332	0.1547	0.0623	-0.0122
1990	0.0640	0.0344	0.1158	0.0679	-0.0087
2000	0.0607	0.0359	0.0923	0.0760	-0.0074
2005	0.0550	0.0352	0.0764	0.0731	-0.0066
2006	0.0537	0.0342	0.0736	0.0724	-0.0064
2007	0.0523	0.0331	0.0710	0.0715	-0.0062
2008	0.0510	0.0319	0.0687	0.0709	-0.0062
2009	0.0495	0.0307	0.0677	0.0679	-0.0059
2010	0.0483	0.0296	0.0667	0.0661	-0.0058
2011	0.0456	0.0270	0.0619	0.0648	-0.0056
2012	0.0447	0.0260	0.0619	0.0630	-0.0056

Table 1: Theil inequality index for the HDI and its components.

2000, although this change (21.6%) is quite small compared to the overall HDI evolution. The situation is even worse for material wellbeing: in 2012 the inequality for this component was higher than it was 32 years before. From 1980 to 2000 income inequality increased by 17.3% and thereafter slightly decreased up to 2012. Therefore, the improvement in global HDI distribution cannot be attributed to material wellbeing, quite the contrary, it has been pushing in the opposite direction.

A positive trend for education is clearly evident. Education inequality decreased from 0.1547 in 1980 to 0.0619 in 2012, a significant decrease of 60% over the study period. Thus, it seems that a reduction in education inequality has been key for the decrease in HDI disparities.

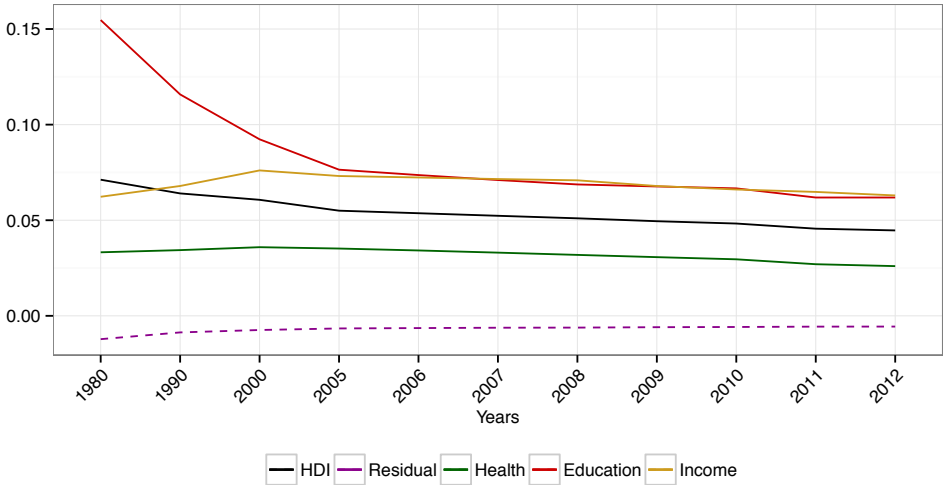

Figure 2: Evolution of the Theil index of the HDI and its components during 1980–2012.

4 Conclusions

Besides use of the HDI as a tool to measure the social achievements of countries, it is natural to wonder whether these achievements are evenly achieved on a global scale. The most intuitive way to determine the level of disparity is to use an inequality index. There are many such indices available and the results may vary depending on the one we use. The Theil index is especially convenient because it allows for a decomposition of the overall HDI inequality into inequalities for the three HDI components, with the same weight for each.

Despite the unavailability of data and the resulting limitations for the number of countries that can be considered, we believe that our results provide a good illustration of HDI evolution, for which disparities have been reduced essentially because of the significant reduction in education inequality.

References

Chakravarty, S. R. (2011). A reconsideration of the tradeoffs in the new human development index. *Journal of Economic Inequality*, 9:471–474.

Herrero, C., Martínez, R., and Villar, A. (2012). A newer human development index. *Journal of Human Development and Capabilities*, 13:247–268.

Klugman, J., Rodríguez, F., and Choi, H.-J. (2011). The HDI 2010: new controversies, old critiques. *Journal of Economic Inequality*, 9:249–288.

Martínez, R. (2011). Inequality and the new human development index. *Applied Economics Letters*, 19:533–535.

Pillarisetti, J. R. (1997). An empirical note on inequality in the world development indicators. *Applied Economics Letters*, 4:145–147.

Ravallion, M. (2011). The human development index: a response to Klugman, Rodriguez and Choi. *Journal of Economic Inequality*, 9:475–478.

——— (2012). Troubling tradeoffs in the human development index. *Journal of Development Economics*, 99:201–209.

Sen, A. (1985). *Commodities and Capabilities*. North-Holland.

Theil, H. (1967). *Economics and Information Theory*. North-Holland.

United Nations Development Programme (2013). *Human Development Report 2013*. United Nations Publications.